# Ph.D. Synopsis /work plan

**On Topic**

# *Study of Nonlinear Fuzzy Integral Equations*

**By**

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# 1) Introduction :

The topic of fuzzy integral equations (FIE) has been developed in recent years. In the first step often including applicable definitions of the fuzzy integrals was followed by introducing FIE and establishing sufficient conditions for the existence of unique solutions to these equations. Finally, numerical algorithms for calculation approximates to these solutions were designed. Prior to discussing fuzzy integral equations and their associated numerical algorithms, it is necessary to present an appropriate brief introduction to preliminary topics such as fuzzy numbers and fuzzy calculus [1]. Concept of fuzzy which firstly introduced by Zadeh [2] [3] leads to definition of the fuzzy number and its implementation in various problems for example fuzzy linear systems [4], fuzzy differential equations [5] [6], fuzzy integral equations [7], and fuzzy integro-differential equations [8]. Fuzzy differential and fuzzy integral equations are very use full tool to model the dynamical systems whose characteristic is biased on vagueness or uncertainty due to partial information about the problem, or due to information which is not fully reliable, or due to inherent imprecision in the language with which the problem is defined, or due to receipt of information from more than one source about the problem which is conflicting .So, the topic fuzzy differential and fuzzy integral equations have been a great area of interest and rapidly growing in recent years. The concept of integration of fuzzy valued functions was first presented by Dubois and Prade [9]. The study of fuzzy integral equations begins with the investigations Suvankar Biswas, Tapan Kumar Roy of Kaleva .Kaleva [10] and Buckley [11] discussed the properties of differentiable fuzzy valued functions. S.Seikkala[12] discusses the initial value problem .The existence and uniqueness theorems for Volterra and Fredholm integral equations involving fuzzy set valued functions of a real variable have been given by Park and Jeong [13][14] The existence theorem of fuzzy Volterra integral equation has been also given by P. V. Subrahmanyam and S. K. Sudarsanam [15]. A numerical algorithm for solving fuzzy Fredholm and Volterra integral equations of the second kind with

arbitrary kernel using a parametric Riemann integral representation has been given by M. Friedman, M. Ming and A. Kandel [16][17][18]. Using a fixed point technique, a quadrature formula for the solution of nonlinear fuzzy Fredholm integral equations have been given by Bica [19]. .There are several numerical methods for fuzzy integral equations .The numerical methods like Admian decomposition, homotopy analysis and homotopy perturbation.There are other technique also like Quadrature rules and Nystrom techniques.

## 2) Review of Literature and Development in the subject (Previous work done in the relevant areas):

Fuzzy integral equations in recent years have been an object of investigation with increasing interest. For more information see the monographs, V. Laksmikantham et.al, Grabisch et al, Garcia-Jacas et al, and; Vilatersana et al.The research papers cited therein.Also many authors has studied existence ,uniqueness and other properties of fuzzy integral equations of more general type or their special forms such as Phil Diamond (2006) , E. Babolian ( 2005 ),F. Mirzaee ( 2015) ,R .Agarwal ( 2012 ) , H. Wang (2018) ,S. Narayanamoorthy (2016), A .Musayev (2018 ) ,V. Torra (2006), A.A. Hamoud (2018), C. Kahraman ( 2016 ) , C. Mnasri (2011) H.Liu (2017), R. P. Agarwal, D. O. Regan(2004), S. Biswas ( 2017), and P.Diamond(2002) etc.

## 3) Objective of Research/ proposed hypothesis:

The purpose of this study is to investigate series solution of various types of linear and nonlinear fuzzy integral equations. Also we study existence, uniqueness, boundedness, stability etc. of fuzzy differential and integro-differential equations.

## 4) Methodology to be adopted:

We use different methods to obtain results such as:

i) Fixed point theorems.

ii) Homotopy analysis methods.

iii) Numerical methods.

iv)Monotone iterative techniques etc.

## 5) Importance of Study/ society applications:

- **International Status :**

Many mathematicians are working in this field, some of them are Agarwal (Singapore), Quim Torra (Spain), Grabisch, Michel (Hungary), César R. García-Jacas (New York), Y. Santur (Spain), Phil Diamond (Orlando, Florida),Esmail Babolian (Iran) , Siroos Mirzaei (Austria), Darren Wang (china), S.Abbasbandy(Iran) etc.

- **National Status :**

Some of the Indian Mathematicians are also working on same line. Few of them are Laxmi Agarwal (New Delhi, India), Prasant Kumar Sahu (Kharagpur, West Bengal), S Tapaswini (New Delhi, India), S. Narayanamoorthy (Coimbatore, Tamil Nadu),) S. P. Sathiyapriya (Coimbatore, Tamil Nadu),) Suvankar Biswas and Tapan Kumar Roy (Howrah, West Bengal) Supratim Biswas(Powai,Mumbai)etc.

- **Significance of study :**

Fuzzy integral equations are important in studying and solving a large proportion of the problems in many topics in applied mathematics, in particular in relation to physics, geographic, medical and biology. Usually in many applications, some of the parameters in our problems are represented by fuzzy number rather than crisp, and hence it is important to develop mathematical models and numerical procedures that would appropriately treat general fuzzy integral equations and solve them.

## 6) Proposed Work plan /Formulation and Structure of study:

| Sr.No | Period | Plan of the work | Targets to be achieved |
|---|---|---|---|
| 1 | First year | Study of the background required for the research topic. | Good command on the subject and clear the course work. |
| 2 | Second year | Collecting the papers and analyze them. | Try to get some new results based on the previous work. |
| 3 | Third Year | Work on some problems to be included in the thesis. | Submit at least two papers. |
| 4 | Fourth Year | Publishing some papers and submitting the Thesis. | Some more papers and Thesis submission. |

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